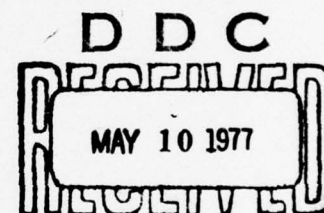ADA039184

STATISTICAL ESTIMATIONS OF GEOLOGICAL MATERIAL MODEL PARAMETERS FROM CYLINDRICAL IN-SITU TEST DATA

University of New Mexico
CERF/J. H. Wiggins Company
Albuquerque, NM 87131

March 1977

Final Report

DDC

MAY 10 1977

C

Approved for public release; distribution unlimited.

DDC FILE COPY

Prepared for
DEFENSE NUCLEAR AGENCY
Washington, DC 20305

AIR FORCE WEAPONS LABORATORY
Air Force Systems Command
Kirtland Air Force Base, NM 87117

This final report was prepared by the University of New Mexico, Albuquerque, New Mexico, under Contract F29601-76-C-0015, Job Order WDNS3320 with the Air Force Weapons Laboratory, Kirtland Air Force Base, New Mexico. Mr. John Thomas (DES) was the Laboratory Project Officer-in-Charge.

When US Government drawings, specifications, or other data are used for any purpose other than a definitely related Government procurement operation, the Government thereby incurs no responsibility nor any obligation whatsoever, and the fact that the Government may have formulated, furnished, or in any way supplied the said drawings, specifications, or other data is not to be regarded by implication or otherwise as in any manner licensing the holder or any other person or corporation or conveying any rights or permission to manufacture, use, or sell any patented invention that may in any way be related thereto.

This report has been reviewed by the Information Office (OI) and is releasable to the National Technical Information Service (NTIS). At NTIS, it will be available to the general public, including foreign nations.

This technical report has been reviewed and is approved for publication.

JOHN N. THOMAS
Project Officer

FOR THE COMMANDER

JAMES M. WARREN
Lt Colonel, USAF
Chief, Survivability Branch

FRANK J. LEECH
Lt Colonel, USAF
Chief, Civil Engineering Research
 Division

DO NOT RETURN THIS COPY. RETAIN OR DESTROY.

(19) REPORT DOCUMENTATION PAGE	READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
AFWL-TR-76-187		

4. TITLE (and Subtitle)	5. TYPE OF REPORT & PERIOD COVERED
STATISTICAL ESTIMATIONS OF GEOLOGICAL MATERIAL MODEL PARAMETERS FROM CYLINDRICAL IN-SITU TEST DATA.	Final Report
	6. PERFORMING ORG. REPORT NUMBER
	76-1259-1

7. AUTHOR(s)	8. CONTRACT OR GRANT NUMBER(s)
Jerrold Isenberg, John D. Collins Bruce Kennedy	F29601-76-C-0015

9. PERFORMING ORGANIZATION NAME AND ADDRESS	10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
University of New Mexico CERF/J. H. Wiggins Company Albuquerque, New Mexico 87131	62704H/WDNS3320/Subtask No. SB144/Work Unit 11

11. CONTROLLING OFFICE NAME AND ADDRESS	12. REPORT DATE
Defense Nuclear Agency Washington, DC 20305	March 1977
	13. NUMBER OF PAGES
	38

14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office)	15. SECURITY CLASS. (of this report)
Air Force Weapons Laboratory (DES) Kirtland Air Force Base, NM 87117	UNCLASSIFIED
	15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Soil Mechanics
Civil Engineering
Geology and Mineralogy

Cylindrical in situ tests

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

A developmental study was performed to aid the AFWL in the identification of soil properties from (CIST) test data. A mathematical algorithm was developed which, using prior estimates of the properties and velocity-time history data from the tests, provided improved estimates of the parameters in the soil model. The algorithm was tested using a computational experiment, i.e., a finite-difference code was used to generate a set of velocity-time history responses based on a predetermined set of parameters. The algorithm was then used to determine a set of parameters based on the data, and comparisons

DD FORM 1 JAN 73 1473   EDITION OF 1 NOV 65 IS OBSOLETE

(cont from p1473A)

20. ABSTRACT

were made with the exact solution. After a number of cycles through the algorithm, a set of parameters was derived which provided satisfactory matching of the velocity-time histories.

Foreword

The work in this study was performed under contract to the University of New Mexico, Civil Engineering Research Facility under USAF contract number F29601-76-C-0015. The project was directed by C.J. Higgins of CERF and Captain G.W. Ullrich of AFWL (DEV). John N. Thomas of AFWL was responsible for programming the partial derivative computation and output format using the AFTON program, and for the preparation of all AFTON runs. The authors must also acknowledge Robert Port, formerly of AFWL, who provided the initial support for performing the study.

SUMMARY

A developmental study was performed to aid the AFWL in the iden-
tification of soil properties from CIST test data. A mathematical
algorithm was developed which, using prior estimates of the properties
and velocity-time history data from the tests, provided improved
estimates of the parameters in the soil model. The algorithm was
tested using a computational experiment, i.e., a finite-difference
code was used to generate a set of velocity-time history responses
based on a predetermined set of parameters. The algorithm was then
used to determine a set of parameters based on the data, and compari-
sons were made with the exact solution. After a number of cycles
through the algorithm, a set of parameters was derived which pro-
vided satisfactory matching of the velocity-time histories.

CONTENTS

ILLUSTRATIONS

TABLES

SECTION I

INTRODUCTION

For several years, AFWL has been conducting a series of CIST (Cylin-
drical In Situ Tests)[1] tests to acquire data on the shock response
of soils and develop models which may be used in making predictions
of response of facilities to ground motion from nuclear weapons
effects. Because of the very high levels of the shock in these tests,
simple linear soil models are not adequate. Consequently, in parallel
to the physical tests, considerable effort has been spent in support
of the modeling of the tests using sophisticated finite difference
codes such as AFTON.

If the assumption is made that the mathematical model in the AFTON
program is valid then, with the right values of the soil parameters
and the proper representation of the initial shock, it should be
possible to duplicate the test response using the computer simulation.
Unfortunately, because of the number of soil parameters involved and
because of their interrelationships, it has been very difficult to
reproduce the response measured in tests by means of the analyst's
soil parameter estimates and computer simulation. The adjustment of
the parameters to obtain the proper response has involved a cut and
try method which required a large effort by individuals who had exten-
sive experience and long exposure to prior data and response, and has
resulted in very limited success.

The problem, as described, made it very difficult to process the large
quantities of CIST data and an alternative approach had to be taken.
It was at this point that the J.H. Wiggins Company, which had devel-
oped a parameter estimation algorithm for NASA[2], proposed a soil
parameter identification algorithm which could be used to systematize
and automate the parameter identification process which had been done
by hand up to that date.

---

1. Davis, S.E., Experimental Data from the Middle Gust and Mixed Com-
   pany CIST Events, Santa Barbara, CA, DNA3151P2, 1 May 1973.

2. Collins, J.D., Hart, G.C., Hasselman, T.K., Kennedy, B., Statisti-
   cal Identification of Structures, AIAA Journal, Vol. 12, No. 2,
   pp. 185-190, February 1974.

Since the parameter identification method was untried in this area of mechanics, it was decided that a test problem be developed which could be used to evaluate the validity of the identification methodology. A problem was designed where the AFTON program was used with a theoretical model to produce a velocity time history of response that the identification algorithm was supposed to match. By using the AFTON output rather than the output from a CIST test, the problem of model validity and test data validity did not have to be considered. The only question was whether the identification algorithm could reproduce the parameters that had been originally used to obtain the velocity time histories.

The following sections of this report contain the results of this experiment, describe the system identification algorithm concept and show how the methodology is implemented on the computer.

The identification algorithm, which is referred to as ESP, is not the standard least squares type of algorithm which is most frequently used in parameter identification. This method, which will be explained later as a Bayesian method, incorporates the judgment of the analyst into the analysis in order to improve the convergence to the proper values.

SECTION II

DEVELOPMENT OF THE ESP MODEL

PARAMETER ESTIMATION

There is a common problem that faces the analyst and the experimentalist in a number of fields. The analyst, from the physics of the problem and past experience, develops a mathematical model to predict the response of the structure or other media. Presumably, given a set of conditions, the model will reflect real life behavior. Unfortunately, when the modeling problem is difficult, the behavior as measured in a physical test frequently does not match satisfactorily the behavior as predicted by the model. This is the dilemma.

There are two problems that can arise in the modeling of the phenomenon: the mathematical model, and the parameters which are used in the model. Both the model and the parameters can be in error; however, if the model is valid then it is the simpler problem to find the correct values of the parameters. If the model is invalid, it is very unlikely that a set of parameters will be found that will produce satisfactory results. In the work performed in this study, the assumption was made that the model was valid. (This in fact was accomplished by developing pseudo test data from a mathematical model and then determining whether the estimation method would produce the proper parameters.) However, in most situations the adequacy of the model can still be challenged. The best solution to this problem is to perform the parameter estimation with several models in order to find that model and parameter set which most adequately matches the real situation (test data).

Assuming now that the model is valid, consider the problem of estimating the parameters to fit the model. The most common parameter estimation method is the least-squares technique. With a simple least squares analysis, one assumes a linear relationship between the variables, say x and y. Using test data gathered from measurements of y as a function of x, one is able to estimate the slope and intercept, i.e., the parameters, for the linear equation. With simple linear models and a quantity of data points, it is a relatively easy

task to obtain the appropriate values of the parameters. However, when the model becomes complex with many parameters and a sparsity of data, the least-squares method becomes inadequate for solving the problem. Variations on the least-squares method, such as weighted least-squares and maximum likelihood are also limited in solving this multiple-parameter, limited-data problem. Consequently, an alternative approach had to be found which could incorporate prior judgments or knowledge of model parameters to aid the parameter identification problem. This alternative approach involves the use of Bayes' Rule, which is explained in the following section.

BAYES' RULE

The fundamental concept embodied in the parameter estimator used in this work is Bayes' rule from probability theory. This formula, originally a mathematical curiosity, has, during the past twenty years, become a basis for modern decision theory.

The equation states:

$$P(A|B) = \frac{P(B|A)P(A)}{P(B|A)P(A) + P(B|\bar{A})P(\bar{A})} \tag{1}$$

where $P(A|B)$ is the probability (conditional) that event A will occur given the occurrence of B, and $P(\bar{A}) = 1 - P(A)$. The equation can also be written in the probability density form

$$P(\ell|y) = \frac{p(y|\ell)}{\int p(y|\ell)p(\ell)d\ell} \, p(\ell) \tag{2}$$

The probability $P(A)$ in equation (2) can be considered as the first estimate of the probability of event A. $P(B|A)$ is the likelihood of the particular B occurring given that A occurred. The $P(A|B)$ is the revision of the probability of A given the particular B. In words, the equation is:

        Prior (first estimate) × Likelihood (comparison of
        data and estimate) → Posterior (revised estimate)

The significance of the formula is that one can guess a set of parameters (the prior), and then make a measurement. The likelihood of that particular measurement, given the prior, is then multiplied by the prior and normalized to give a new estimate of the set of parameters (the posterior).

Bayes' rule is the only formal place in probability and statistical theory where human judgment can be combined with measurement to produce a better answer. This characteristic is particularly desirable when measured data are meager and past experience can be used to guide it in the right direction. If there is a great abundance of data, Bayes' rule usually loses its significance.

In the application to soil model parameters, the analyst is seeking a "posterior model" of the hydrostat, the failure surface, and Poisson's ratio. Hopefully this posterior model will reflect the actual dynamic behavior of the soil in the test and can be considered to be the best final estimate of the actual soil model. To get the posterior model, the analyst first estimates the parameters of the hydrostat and the failure surface (the prior) and then within the estimation algorithm establishes the probability of obtaining the particular test results with this model (the likelihood).

In this case, the estimated soil model parameters (hydrostat, failure surface, and Poisson's ratio) and their uncertainties form the prior. The procedure in the following development will show how the prior statistical model is combined with the test measurements to obtain the posterior model. The posterior model is, ideally, the identified system.

EXPANSION AROUND A REFERENCE VALUE

The shock response of a soil system may be represented like any other mathematically well-behaved functions in terms of a Taylor's series. For m mathematical functions $(f_1, f_2, \ldots f_m)$ of n variables $(\alpha_1, \alpha_2, \ldots \alpha_n)$ this yields in vector form:

$$\{f(\alpha)\} = \begin{Bmatrix} f_1(\alpha_1, \ \alpha_2, \ \dots \ \alpha_n) \\ f_2(\alpha_1, \ \alpha_2, \ \dots \ \alpha_n) \\ \vdots \\ f_m(\alpha_1, \ \alpha_2, \ \dots \ \alpha_n) \end{Bmatrix} \tag{3}$$

In terms of soil mechanics this means that for a given set of initial conditions the state of stress, the velocity and the strain of the material are functions, $\{f(\alpha)\}$, of the constitutive parameters of the material, $(\alpha_1, \ \alpha_2, \ \alpha_3, \dots)$. For example, the peak particle velocities in the soil at given points, for a set of initial conditions, are functions of quantities such as mass density, Poisson's ratio and wave speeds. For this example the number of responses is the total number of locations at which the peak velocity is measured.

The system of equations can be expanded into a Taylor's series around a reference vector $\{\alpha_p\}$

or

$$\begin{Bmatrix} \alpha_{1,p} \\ \alpha_{2,p} \\ \vdots \end{Bmatrix}$$

$$\{f(\alpha)\} = \{f(\alpha_p)\} + \left[\frac{\partial f(\alpha)}{\partial(\alpha)}\right] \{\alpha - \alpha_p\} + O\left(\{\alpha - \alpha_p\}\{\alpha - \alpha_p\}^T\right) \tag{4}$$

where O denotes the order of magnitude of the truncation error

$$\{f(\alpha_p)\} \equiv \begin{Bmatrix} f_1(\alpha_{1,p}, \ \alpha_{2,p}, \ \dots \ \alpha_{n,p}) \\ f_2(\alpha_{1,p}, \ \alpha_{2,p}, \ \dots \ \alpha_{n,p}) \\ \vdots \\ f_m(\alpha_{1,p}, \ \alpha_{2,p}, \ \dots \ \alpha_{n,p}) \end{Bmatrix}$$

and

$$
\left[\frac{\partial f(\alpha)}{\partial(\alpha)}\right] \equiv
\begin{bmatrix}
\dfrac{\partial f_1}{\partial \alpha_1} & \dfrac{\partial f_1}{\partial \alpha_2} & \cdots & \dfrac{\partial f_1}{\partial \alpha_n} \\[2ex]
\dfrac{\partial f_2}{\partial \alpha_1} & \dfrac{\partial f_2}{\partial \alpha_2} & \cdots & \dfrac{\partial f_2}{\partial \alpha_n} \\[2ex]
\dfrac{\partial f_m}{\partial \alpha_1} & \dfrac{\partial f_m}{\partial \alpha_2} & \cdots & \dfrac{\partial f_m}{\partial \alpha_n}
\end{bmatrix}_{\{\alpha\} = \{\alpha_p\}}
$$

The Taylor's series expansion shown here is valid when $f(\alpha)$ possesses continuous first and second derivatives. Indeed the truncation error represented by the term $O\left(\{\alpha - \alpha_p\}\{\alpha - \alpha_p\}^T\right)$ is zero when the statistical model is linear. It is fortunate that most functions governing physical phenomena are expandable into a Taylor's series, thus permitting the above operations to be carried out.

It should be noted that the shock response of a soil system is always a nonlinear function of the soil state vector, $\{\alpha\}$. The assumption of the truncated Taylor's series shown above is appropriate when the changes in the soil parameter $\{\alpha - \alpha_p\}$ are small. Obviously as these changes approach zero, the truncation error will approach zero even faster and the formulation given by the truncated series becomes, in the limit, exact. Thus the repeated application of a seemingly linear statistical model can be employed to solve a nonlinear statistical model, by ignoring the truncation error.

By dropping the truncation term, equation (4) will be simplified to the approximation

$$
\{\Delta f\} = \left[\frac{\partial f(\alpha)}{\partial(\alpha)}\right]\{\Delta \alpha\} \tag{5}
$$

where $\quad \{\Delta f\} = \{f(\alpha) - f(\alpha_p)\}$

and $\quad \{\Delta \alpha\} = \{\alpha - \alpha_p\}$

-11-

If the number of responses, m, is the same as the number of parameters, n, and if the matrix

$$\left[\frac{\partial f(\alpha)}{\partial(\alpha)}\right]$$

is nonsingular, equation (5) can be readily used to solve for $\{\Delta\alpha\}$. However, m and n are usually not equal and estimation techniques must be resorted to in order to determine $\{\Delta\alpha\}$.

STATISTICAL DEFINITIONS

Before outlining the statistical identification process, it is imperative that statistical concepts be reviewed.

$E(x)$ or $\bar{x}$ is the expected or mean value of the random variable x. Since E is a linear operator, the expected value of a vector $\{x\}$ can be readily computed to be

$$E\left(\{x\}\right) = E\left\{\begin{matrix}x_1\\x_2\\\vdots\end{matrix}\right\} = \left(\left\{\begin{matrix}E(x_1)\\E(x_2)\\\vdots\end{matrix}\right\}\right) = \left\{\begin{matrix}\bar{x}_1\\\bar{x}_2\\\vdots\end{matrix}\right\} = \{\bar{x}\} \tag{6}$$

The variance (or standard deviation squared) of a random variable is

$$Var(x_i) = \sigma^2_{x_i} = E\left((x_i - \bar{x}_i)^2\right) \tag{7}$$

The covariance of two random variables is

$$Cov(x_i x_j) = \sigma_{x_i x_j} = E\left((x_i - \bar{x}_i)(x_j - \bar{x}_j)\right) = \rho_{ij}\sigma_i\sigma_j \tag{8}$$

where ρ_{ij} is defined as the correlation coefficient for random variables x_i and x_j and can be shown to have a range of $-1 \le \rho_{ij} \le 1$.

The variances and covariances of a vector are obtained by taking the expected value of the outer product of the vector

$$E\left(\{x - \bar{x}\}\{x - \bar{x}\}^T\right) = E\left(\{\Delta x\}\{\Delta x\}^T\right) = [S_{xx}]$$

$$= \begin{bmatrix} E\left((x_1 - \bar{x}_1)^2\right) & E\left((x_1 - \bar{x}_1)(x_2 - \bar{x}_2)\right) & \cdot \\ E\left((x_1 - \bar{x}_1)(x_2 - \bar{x}_2)\right) & E\left((x_2 - \bar{x}_2)^2\right) & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{bmatrix}$$

$$= \begin{bmatrix} \sigma_{x_1}^2 & \sigma_{x_1 x_2} & \cdot \\ \sigma_{x_1 x_2} & \sigma_{x_2}^2 & \cdot \\ \cdot & \cdot & \cdot \end{bmatrix} = \begin{bmatrix} \sigma_{x_1}^2 & \rho_{12}\sigma_{x_1}\sigma_{x_2} & \cdot \\ \rho_{12}\sigma_{x_1}\sigma_{x_2} & \sigma_{x_2}^2 & \cdot \\ \cdot & \cdot & \cdot \end{bmatrix}$$

If the vector $\{\Delta z\} = \{z - \bar{z}\}$ is related to $\{\Delta x\}$ by the linear transformation

$$\{\Delta z\} = [c]\{\Delta x\} \tag{10}$$

then

$$\begin{aligned} [S_{zz}] &= E\left(\{\Delta z\}\{\Delta z\}^T\right) \\ &= E\left([c]\{\Delta x\}\{\Delta x\}^T[c]^T\right) \\ &= [c]E\left(\{\Delta x\}\{\Delta x\}^T\right)\{c\}^T \\ &= [c][S_{xx}][c]^T \tag{11} \end{aligned}$$

-13-

DERIVATION OF STATISTICAL SYSTEM IDENTIFICATION EQUATIONS

Earlier in this section an expression was developed which related
the response of a soil system to its soil parameters. The relation-
ship will now be used to solve the inverse problem of determining
the values of the soil parameters when the response is specified.
The derivation procedure used is based on that of a modified
weighted least squares.

The cases when no errors are present in the experimental error will
be treated first. If the mathematical function $f(\alpha)$ is the true
representation of the physical phenomenon, then the measured response
f_r will be equal to $f(\alpha)$ appearing in equation (3). Under these con-
ditions, equation (5) can now be recast as

$$\{F\} = [T]\{A\} \tag{12}$$

where

$$\{F\} = \{f_r - f(\alpha_p)\}$$

and

$$\{A\} = \{\alpha - \alpha_p\}$$

It is important to determine the statistical behavior of the random
variables in equation (12). It can be assumed that the prior assumed
values of the soil parameters are distributed with mean

$$E(\{\alpha_p\}) = \{\bar{\alpha}\} \tag{13}$$

The covariance matrix of $\{\alpha\}$ which must be specified by the soil-
analyst is

$$[S_{\alpha\alpha}] = E\left(\{\alpha - \alpha_p\}\{\alpha - \alpha_p\}^T\right) \tag{14}$$

From equation (13), the mean and covariance matrix of $\{A\}$ are

$$E(\{A\}) = \{0\} \tag{15}$$

and

$$[S_{AA}] = E(\{A\}\{A\}^T) = [S_{\alpha\alpha}] \tag{16}$$

If in equation (12), $\{A\}$ is defined by a multivariate normal distribution, then $\{F\}$ through the linear transformation is also normally distributed with mean

$$E(\{F\}) = 0$$

and covariance

$$[S_{FF}] = E(\{F\}\{F\}^T) = E([T]\{A\}\{A\}^T\{T\}^T) \qquad (17)$$

$$= [T][S_{\alpha\alpha}][T]^T$$

The above relation must now be amended to account for the error in the measurements, $\{\varepsilon\}$. These errors are independent of the uncertainty resulting from the soil parameter estimates, but do not appear in equation (12). To account for this measurement error, the vector $\{\varepsilon\}$ is added to the right-hand side of equation (12), giving

$$\{F\} = [T]\{A\} + \{\varepsilon\} \qquad (18)$$

The vector $\{\varepsilon\}$ being unbiased has a zero mean and a covariance $[S_{\varepsilon\varepsilon}]$ specified from the test measurements. The mean of $\{F\}$ is still zero, but the covariance is

$$[S_{FF}] = E(\{F\}\{F\}^T)$$

$$= E\left(([T]\{A\} + \{\varepsilon\})([T]\{A\} + \{\varepsilon\})^T\right)$$

$$= [T][S_{\alpha\alpha}][T]^T + [S_{\varepsilon\varepsilon}] \qquad (19)$$

The covariance of $\{F\}$ and $\{A\}$ which is required later in the development is

$$[S_{FA}] = E(\{F\}\{A\}^T)$$

$$= E\left(([T]\{A\} + \{\varepsilon\})\{A\}^T\right)$$

$$= E([T]\{A\}\{A\}^T + \{\varepsilon\}\{A\}^T)$$

$$= [T][S_{AA}] = [T][S_{\alpha\alpha}] \tag{20}$$

where the expected value of $\{\varepsilon\}\{A\}^T$ is zero since the vectors $\{\varepsilon\}$ and $\{A\}$ are statistically independent.

The purpose of this process is to find a best estimator of $\{A\}$ based on the measured responses $\{f_r\}$ and the prior estimates of the soil parameters $\{\alpha_p\}$ along with their associated errors. We are therefore seeking an equation of the form

$$\{A^*\} = [G]\{F\} \tag{21}$$

The matrix $[G]$ will now be defined in such a way as to minimize the variance of the difference between the true value of $\{A^*\}$ and its estimated value. Quantitatively, the equation

$$[Q] = E[\{A^* - A\}\{A^* - A\}^T] \tag{22}$$

is minimized with respect to $[G]$. First substitute equation (21) into (22) yielding

$$[Q] = E\left[([G]\{F\} - \{A\})([G]\{F\} - \{A\})^T\right]$$

$$= [G][S_{FF}][G]^T - [G][S_{FA}] - [S_{FA}]^T[G]^T + [S_{AA}] \tag{23}$$

Next, taking the variation of this with respect to $[G]$, we obtain

$$0 = [\delta G]([S_{FF}][G]^T - [S_{FA}]) + ([G][S_{FF}] - [S_{FA}]^T)[\delta G]^T \tag{24}$$

This yields

$$[G] = [S_{FA}]^T[S_{FF}]^{-1}$$

$$\{A^*\} = [S_{FA}]^T[S_{FF}]^{-1}\{F\} \tag{25}$$

which after substitution from equations (12), (19) and (20) becomes

-16-

$$\{\alpha*\} = \{\alpha_p\} + [S_{\alpha\alpha}][T]^T([T][S_{\alpha\alpha}][T]^T + [S_{\epsilon\epsilon}])^{-1} \quad \{f_r - f(\alpha_p)\}$$

(26)

The covariance of the estimate of $\{\alpha*\}$ is

$$[S^*_{\alpha\alpha}] = E[\{A* - A\}\{A* - A\}^T]$$

$$= [S_{AA}] - [S_{FA}]^T[S_{FF}]^{-1}[S_{FA}]$$

$$= [S_{\alpha\alpha}] - [S_{\alpha\alpha}][T]^T([T][S_{\alpha\alpha}][T]^T + [S_{\epsilon\epsilon}])^{-1} [T][S_{\alpha\alpha}]$$

(27)

ESTIMATOR OPERATION

Figure 1 describes the sequence of operations used in the soil model parameter identification process. Here $\{t\}$ and $\{U\}$ are the response variables contained in $\{f\}$. If the relation between the residual in the observation vector $\{F\}$ and the state vector $\{\alpha\}$ were indeed linear, the method would yield a best estimate in one step without iteration. However, the responses that are computed are usually linked to the state vector in some nonlinear fashion. Consequently, a solution must be obtained by repeated iteration using the successive approximation method.

As shown in figure 1, the experimentalist supplies the ESP (Estimation of Soil Parameters) program with measured responses and uncertainties from CIST experiments. The analyst supplies the program with an estimate of the state vector of soil parameters along with their uncertainties. The procedure is then cycled between the AFTON and ESP programs until convergence is obtained.

By far the most lengthy chain in the procedure just described is the calculation of the partial derivative matrix. It should be noted that it is the number of parameters rather than responses that dictates the computational time involved for this calculation. Thus, for example, the number of computations used for the 12 parameters studied here was 13 (1 for the unperturbed state and 12 for the individually perturbed states).
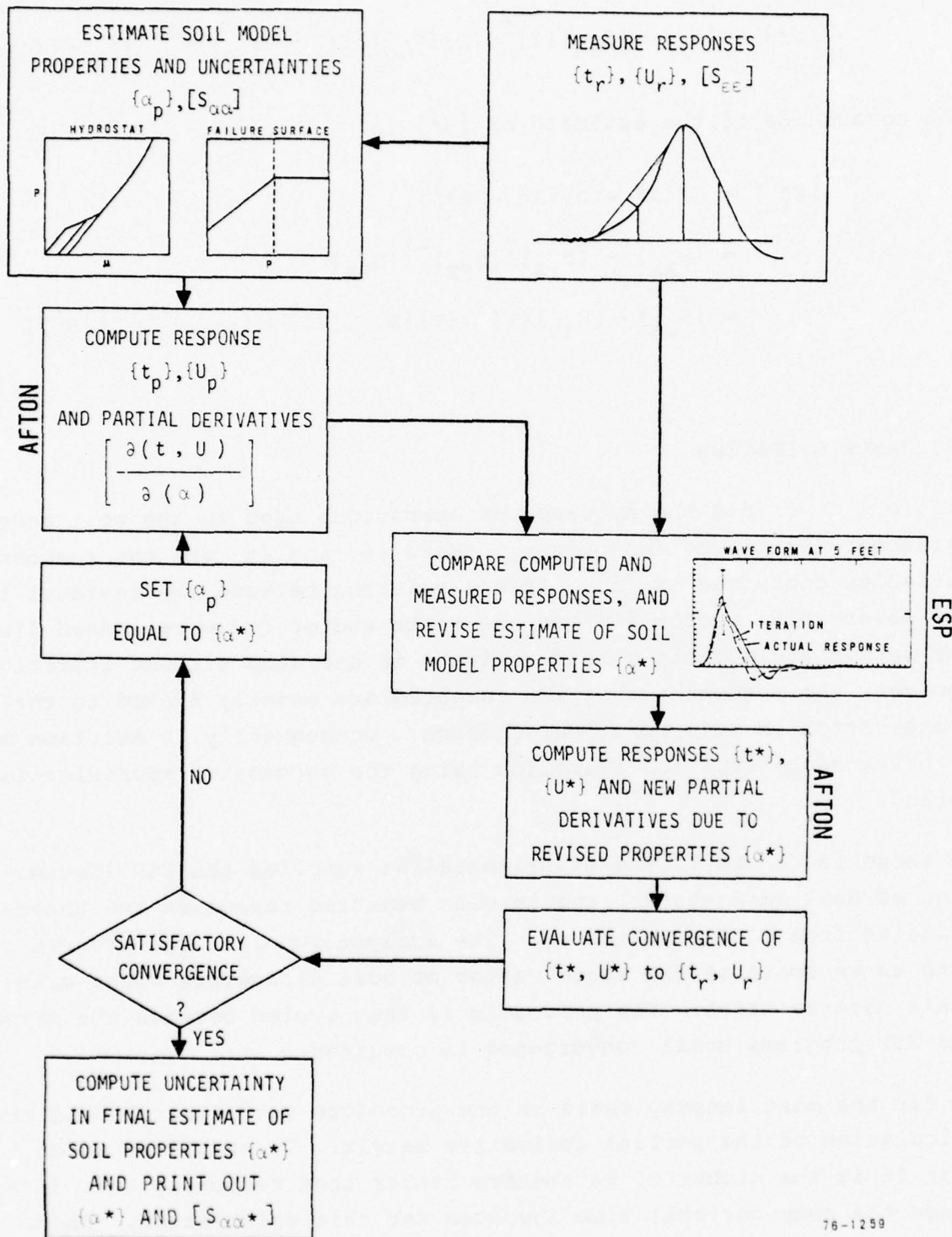
Figure 1.   Sequence of Operations of the
Soils Identification Process

OBSERVATIONS ON THE ESTIMATOR

The procedure used in the ESP program continuously updates the prior
estimate while using the uncertainties of the initial prior estimate
as originally specified by the analyst. The procedure can be im-
proved by minimizing the weighted square of the difference between
the initial prior value and the best estimate. In such a sequence
three state vectors would be involved; the initial prior estimate,
the iterative estimate and the unknown best estimate. The sequence
would iterate until the iterative estimate and the unknown best esti-
mate are equal. Obviously, the effect of this suggested change to
the ESP program must be studied before any conclusions can be drawn.

It should be noted that the value of the best estimate obtained in
the ESP program not only depends on the experimental response and
prior estimates, but also on their associated uncertainties. When
the uncertainty of the experimental response is large, the value of
the best estimate will be close to the prior. Conversely, when the
uncertainty of the prior estimate is large, the value of the best
estimate will be such that the calculated response will be close to
the experimental response. Thus, it is important that the uncertain-
ties be chosen with care.

SECTION III

TEST OF THE ESP PROGRAM

BACKGROUND

In order to obtain in situ material properties for soils and rock for use in predicting crater and ground shock due to nuclear attack, AFWL has conducted CIST (Cylindrical In Situ Test) experiments in the U.S., Alaska and in the Pacific. In these tests PETN explosive (more commonly known as Primacord), with a weight of 5 pounds per linear foot, is inserted into a 2-foot diameter hole to a depth ranging from 40 to 80 feet. The soil layers around the hole are instrumented with accelerometers which measure acceleration as a function of time in the CIST event. A typical example of a velocity waveform generated by integrating the acceleration with time is depicted in figure 2. Velocity response is of particular interest because it can be readily interpreted and compared to the predicted waveform generated by the mathematical finite difference model. For a given location, the significant responses which can be used to describe the waveform are the arrival time, bow time, bow velocity, peak time, peak velocity, and time to 1/2 peak. The arrival time is arbitrarily defined to be the time when the velocity rises to 5% of the peak velocity. (The random fluctuations shown prior to arrival represent background noise.) The bow point is defined as the maximum perpendicular distance of the waveform from a line connecting the arrival point to the peak point. The 1/2 peak time is defined at that time when the velocity falls to half the peak velocity.

The simulation of ground response at AFWL is executed by means of a set of computer codes known as AFTON. This name specifies programs used to solve transient continuum motion problems. To date, three AFTON codes have been developed: AFTON 1 is used to solve one-dimensional problems, AFTON 2P simulates flow in plane-symmetric systems, and AFTON 2A is used to solve flow in axisymmetric systems where radius and axial position are the space variables. AFTON 2A is the program which models the CIST experiment described above.
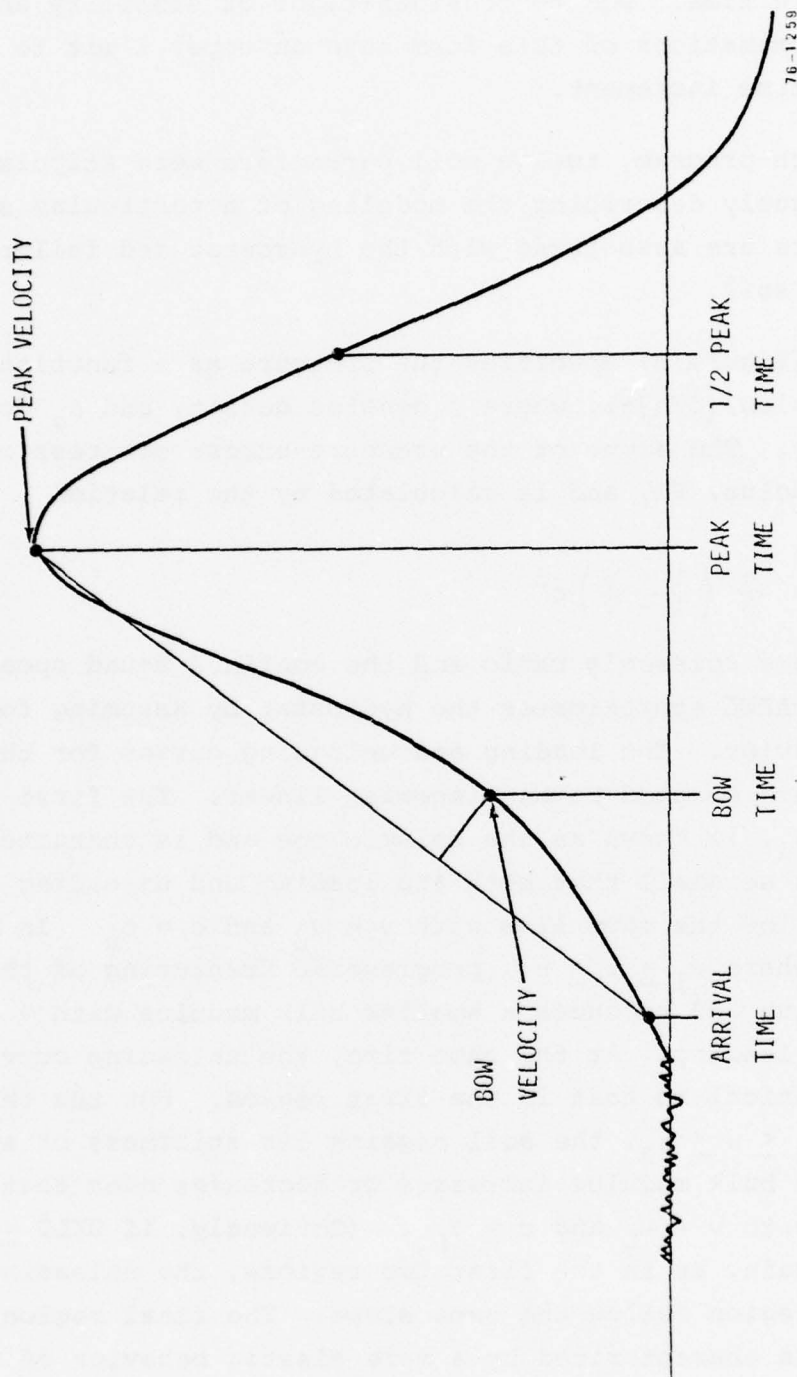
Figure 2. Representative Waveform

Briefly, AFTON 2A expresses the conservation of mass, momentum and energy in a set of explicit finite difference approximations that march forward in time. Due to considerations of stability and convergence, approximations of this form have an upper limit to the size of their time increment.

In this research program, twelve soil parameters were stipulated by AFWL as uniquely describing the modeling of a particular soil. These parameters are associated with the hydrostat and failure surface of the soil.

The hydrostat (figure 3) specifies the pressure as a function of excess compression, $(\rho/\rho_0)-1$, where ρ denotes density and ρ_0 denotes initial density. The slope of the pressure-excess compression curve is the bulk modulus, BK, and is calculated by the relation

$$ BK = \frac{\rho_0}{3} \left(\frac{1 + \nu}{1 - \nu} \right) c^2 \tag{23} $$

where ν and c are Poisson's ratio and the confined sound speed, respectively. AFWL approximates the hydrostat by assuming four regions of behavior. The loading and unloading curves for the first three regions are assumed to be piecewise linear. The first region, where $0 \leq \mu \leq \mu_1$, is known as the seismic toe and is characterized by disturbances so small that both the loading and unloading curves essentially follow the same line with $\nu = \nu_u$ and $c = c_u$. In the second region where $\mu_1 \leq \mu \leq \mu_2$, progressive fracturing of the soil induces softening and produces a smaller bulk modulus with $\nu = \nu_L$ and $c = c_{L_1}$ during loading. At the same time, the unloading curve has a slope, BK, identical to that in the first region. For the third region where $\mu_2 \leq \mu \leq \mu_3$, the soil regains its stiffness or softens further and the bulk modulus increases or decreases over that of the second region with $\nu = \nu_L$ and $c = c_{L_2}$. (Obviously, if BKL2 $\leq$ BKL1, $C_{L_2} \leq C_{L_1}$.) Again, as in the first two regions, the unloading curves for the third region follow the same slope. The final region is that of lockup and is characterized by a more elastic behavior of the soil. In this region, the bulk modulus is initially determined by $\nu = \nu_u$ and $c = c_z$, but the slope rises exponentially and eventually attains an asymptotic value of 10^7 psi. However, in unloading, the curve follows

-22-
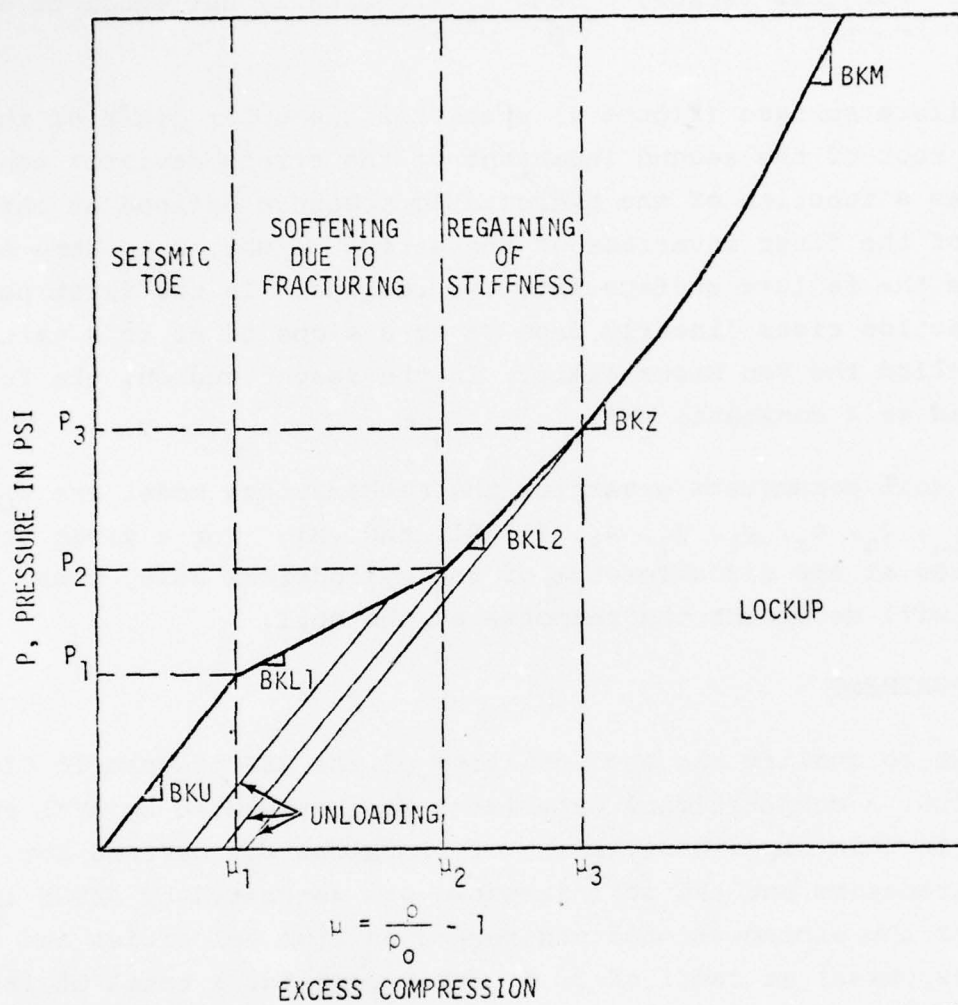
Figure 3. Hydrostat of Soil

76-1259

-23-

the loading curve back down to $\mu = \mu_3$ and then follows the same
unloading slope as in the other regions. Therefore, if c_u and c_z
are not equal, there will be a discontinuity in the bulk modulus at
$\mu = \mu_3$. For this reason, c_u and c_z are usually set equal to each
other.

The failure surface (figure 4) specifies the upper limit of the
square root of the second invariant of the stress deviator tensor,
$\sqrt{J_2'}$, as a function of the hydrostatic pressure defined as one-
third of the first invariant of the stress tensor, J_1. Here AFWL
divides the failure surface into two regions. In the first part,
the function rises linearly from Y1 at a slope of S1 to a value of
VM1, called the Von Mises limit. In the second region, the function
is taken as a constant, VM1.

The 12 soil parameters governing the mathematical model are ν_L, ν_u,
c_{L_1}, c_{L_2}, c_u, c_z, p_1, p_2, μ_3, Y1, S1, and VM1. For a given initial
condition at the circumference of the cylindrical hole, these para-
meters will determine the response of the soil.

THE EXPERIMENT

In order to qualify the applicability of the ESP program to CIST
and AFTON, a computational experiment was formulated by AFWL and
CERF. In this experiment, a set of 12 values was defined for the
soil parameters and the soil response was generated by AFTON in
terms of the aforementioned six responses (two velocities and four
time responses) at radii of 3, 5, and 8 feet for a total of 18
responses. For this calculation the boundary condition simulating
the explosion was a pressure pulse with a time-dependent exponential
decay ($p = 7000\ e^{-400t}$ [psi]). The 18 responses were then supplied
to the J. H. Wiggins Company as experimental measurements, along
with the estimated error in these measurements. Then an initial
estimate of values of the soil model parameters was made by AFWL
(with no knowledge of the actual value of the parameters) and
supplied to the Wiggins Company as the prior estimate of soil para-
meters along with the estimated error in these parameters. The task
of the Wiggins Company was to determine best estimates of the soil
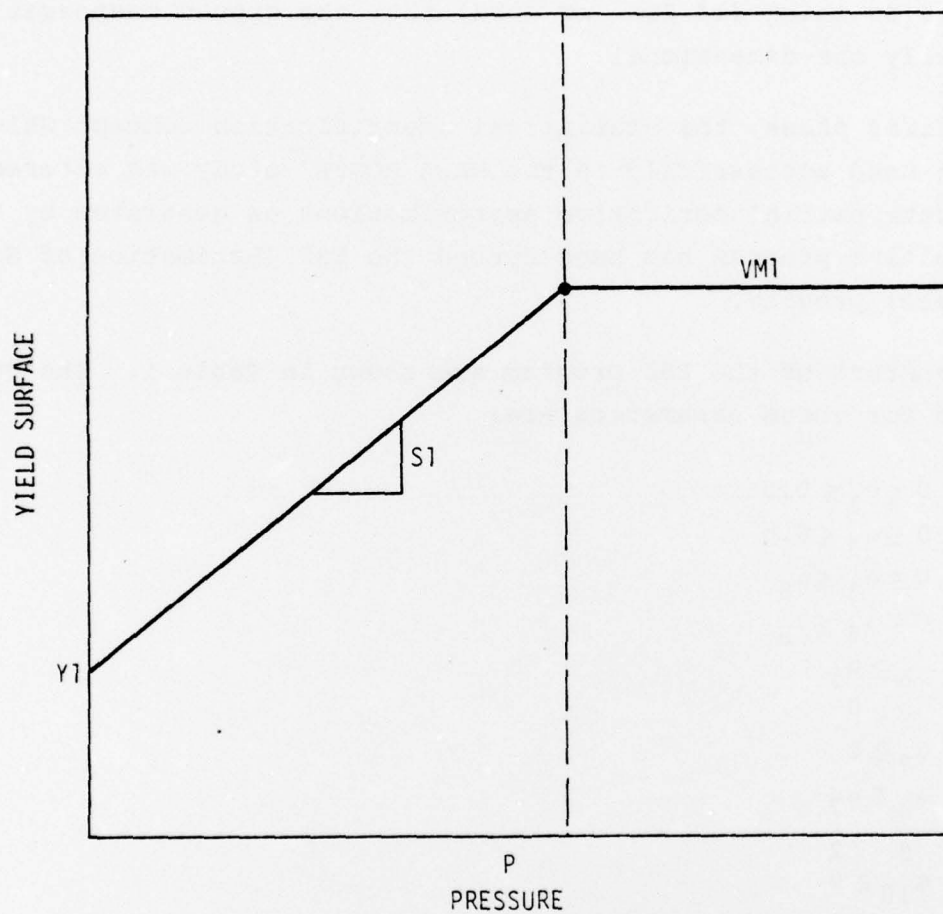
Figure 4. Failure Surface of Soil ($\sqrt{J_2'} \leq$ Yield Surface)

parameters that yielded responses that were statistically within the bounds of the measured responses, by using the prior estimate supplied by AFWL.

For this computational experiment the mathematical model was assumed to be one-dimensional with respect to radius. The computed values and the perturbations (required for the partial derivative approximations) were computed in one AFTON 13-layer run. For this run each layer was so thick (14 feet or more) that the ground responses were essentially one-dimensional.

In the first phase, the statistical identification concept which had been used successfully in the NASA MOUSE[3] study was altered to incorporate partial derivative approximations as generated by AFTON. The resultant program has been dubbed the ESP (Estimation of Soil Parameters) program.

The parameters of the ESP program are shown in Table 1. The constraints for these parameters are:

1. $0 \leq \alpha_1 \leq 0.5$
2. $0 \leq \alpha_2 \leq 0.5$
3. $0 \leq \alpha_3 \leq \alpha_5$
4. $0 \leq \alpha_4 \leq \alpha_5$
5. $\alpha_6 \geq \alpha_5$
6. $\alpha_5 > 0$
7. $\alpha_7 \geq 0$
8. $\alpha_8 \leq \alpha_7$
9. $\alpha_9 \geq \mu_2$
10. $\alpha_{10} \geq 0$
11. $\alpha_{11} \geq 0$
12. $\alpha_{12} \geq \alpha_{10}$

---

3. Collins, J. D., Hart, G. C., Hasselman, T. K., Kennedy, B., _Model Optimization Using Statistical Estimation_, J. H. Wiggins Company, California, 1973   -26-

Table 1. PARAMETERS OF THE ESP PROGRAM

Parameter	Alternate Name	Units
ν_L	α_1	Dimensionless
ν_u	α_2	Dimensionless
C_{L1}	α_3	ft/sec
C_{L2}	α_4	ft/sec
C_u	α_5	ft/sec
C_z	α_6	ft/sec
P_1	α_7	psi
P_2	α_8	psi
μ_3	α_9	Dimensionless
Y_1	α_{10}	psi
S_1	α_{11}	Dimensionless
VM1	α_{12}	psi

where

$$\mu_2 = \frac{1.5 \times 10^6}{(30.48)^2} \; \frac{1 - \alpha_1}{1 + \alpha_1} \left(\frac{\alpha_8 - \alpha_7}{\alpha_3^2} + \frac{\alpha_7}{\alpha_5^2} \right) \qquad (29)$$

These constraints arise either from classical elasticity-plasticity or from assumptions in the finite difference model.

A schematic of the computational sequence is presented in figure 5. After AFWL defined the true parameters and provided the Wiggins Company with the initial estimate of the parameters along with the correct response, the sequence entered a computational loop. In this loop, the Wiggins Company, using the statistical identification process, calculated a modified set of parameters and tested them for convergence. If convergence had not been obtained, AFWL re-ran AFTON to obtain the response and partial derivative matrix associated with the modified set of parameters.

RESULTS

The ESP program cycled the test case through six iterations and the results indicated the pertinent soil parameters to be converging to the statistical limits of the true parameter vector. Table 2 summarizes the results obtained for the responses for each iteration. This convergence is shown graphically in figure 6, where the responses are depicted for stations at 3, 5 and 8 feet from the point of detonation. As the waveforms show, the comparison between results for the fourth iteration and the actual response is especially good for the frontal portion of the response. (Although a total of 6 iterations were carried out, the waveforms for the last 2 iterations were received at the Wiggins Company after the illustrations had been completed.) This is to be expected since most of the measurements provided to the ESP program were taken from this part of the waveform.

Table 3 summarizes the results for the soil parameters for each iteration. The last row indicates the number of the constraints violated. α_1 violated its constraint in the first and third

-28-
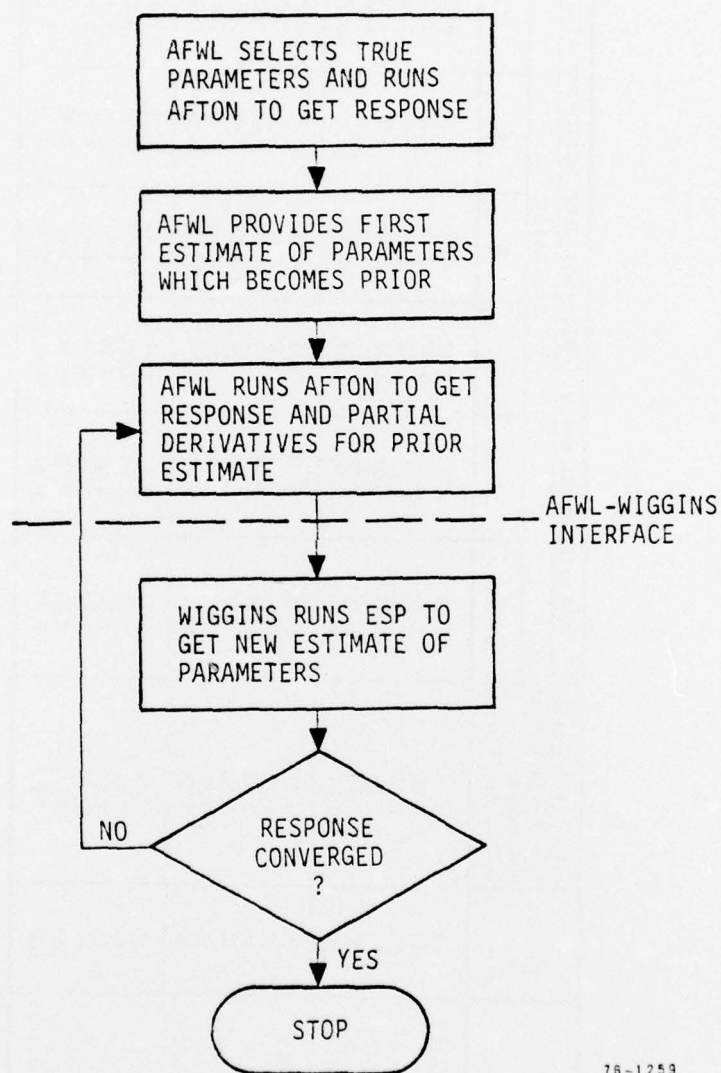
Figure 5. Flow Diagram of Computational Sequence between AFWL and Wiggins Company

Table 2. SUMMARY OF RESPONSES FOR EACH ITERATION

			Values of Response from True Values of Parameters (Experimental Value)	Calculated Values of Responses from Estimates of Parameters						
	Response	Percent Estimate of Error		Iteration						
				Initial	1	2	3	4	5	6
3 feet	arrival time (msec)	10	0.38	0.31	0.41	0.39	0.40	0.39	0.39	0.38
	bow time (msec)	10	0.66	0.53	0.61	0.92	0.84	0.80	0.82	1.46
	peak time (msec)	10	1.94	0.96	1.27	1.52	1.54	1.65	1.80	2.10
	1/2 peak time (msec)	10	3.95	1.19	1.61	2.03	2.80	3.07	3.31	3.66
	bow velocity (m/sec)	20	1.45	1.22	1.99	2.88	2.54	2.05	1.95	8.50
	peak velocity (m/sec)	20	11.41	5.83	12.10	11.64	12.15	11.35	10.91	11.06
5 feet	arrival time (msec)	10	0.77	0.58	0.80	0.74	0.81	0.77	0.80	0.74
	bow time (msec)	10	1.32	0.97	1.63	1.78	1.61	1.59	1.72	1.52
	peak time (msec)	10	2.93	1.44	2.28	2.62	2.59	2.65	2.80	2.77
	1/2 peak time (msec)	10	4.50	1.69	2.71	3.14	3.35	3.66	3.87	4.17
	bow velocity (m/sec)	50	0.88	0.83	1.33	1.39	1.40	1.35	1.42	1.23
	peak velocity (m/sec)	50	5.98	2.84	5.74	4.85	6.64	6.68	6.54	5.88
8 feet	arrival time (msec)	10	1.24	1.03	1.21	1.21	1.41	1.35	1.41	1.34
	bow time (msec)	10	2.44	1.74	2.23	2.23	3.03	2.93	3.17	2.77
	peak time (msec)	10	3.70	2.20	3.58	3.46	3.91	3.91	4.17	4.00
	1/2 peak time (msec)	10	5.07	2.40	3.95	4.22	4.45	4.50	4.75	4.96
	bow velocity (m/sec)	100	0.69	0.64	0.59	0.76	1.02	0.99	1.03	0.91
	peak velocity (m/sec)	100	3.63	1.25	0.82	1.07	3.05	3.29	3.23	3.75

WAVEFORM AT 3 FEET

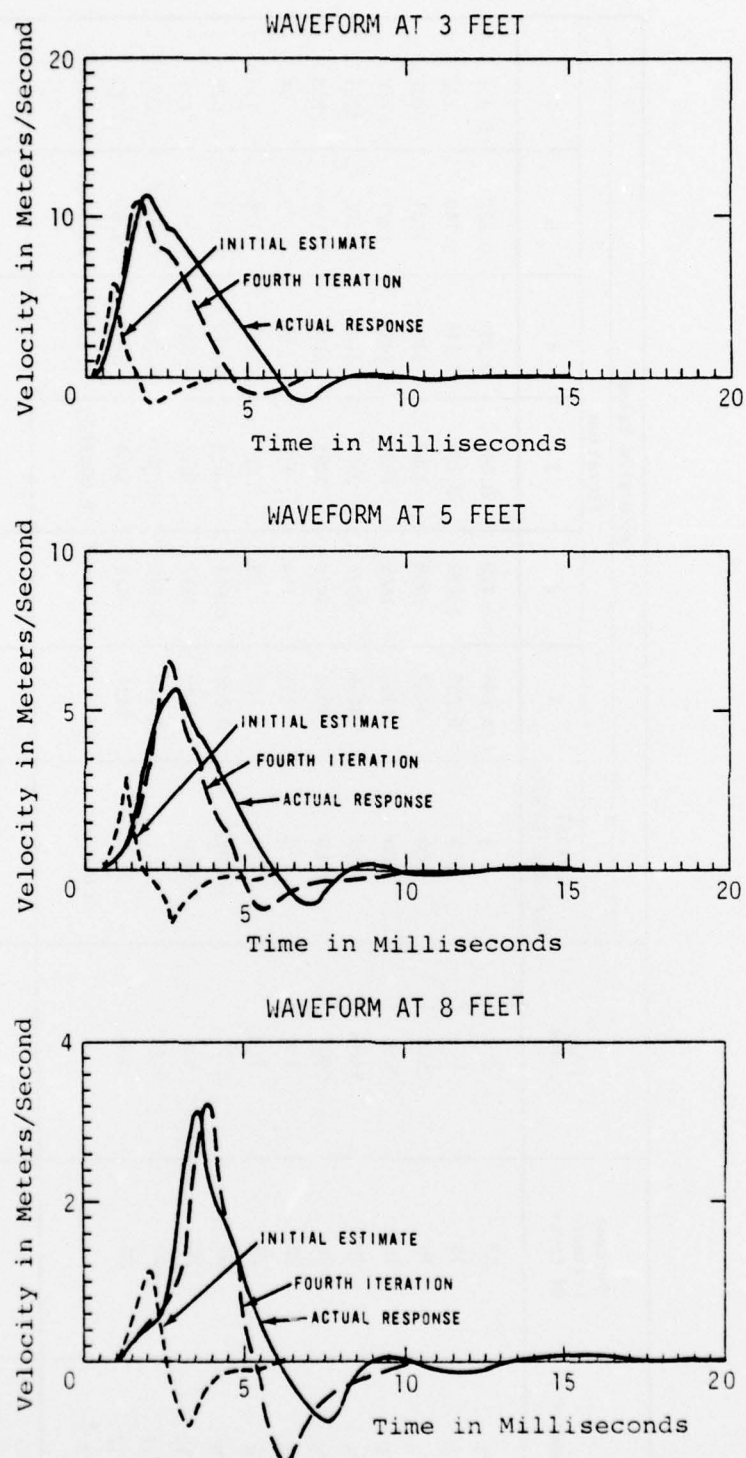WAVEFORM AT 5 FEET

WAVEFORM AT 8 FEET

76-1259

Figure 6. Comparisons of Waveforms of Identification
Parameters with Actual Waveforms

Table 3. SUMMARY OF PARAMETRIC VALUES FOR EACH ITERATION

Parameter	True Value	Percent Estimate Of Error	Initial Estimate (prior)	Calculated Values Iteration 1	2	3	4	5	6
a_1	0.3	33	0.3	0.595	0.428	0.486	0.399	0.372	0.331
a_2	0.3	33	0.3	0.299	0.136	0.181	0.146	0.143	.132
a_3	2500	15	3500	3462	3458	3364	3382	3131	2987
a_4	2500	15	3500	1368	1645	1959	2049	1921	2278
a_5	5000	10	5000	4066	4099	3927	4181	4026	4331
a_6	5000	25	5000	3078	3422	3089	3196	3108	2858
a_7	100	20	125	135	134	112	91	89	94
a_8	100	20	125	123	79	121	131	126	118
a_9	0.05	300	0.0062	0.0087	0.012	0.013	0.014	0.019	0.026
a_{10}	500	20	3000	1549	1317	1012	874	843	621
a_{11}	0.01	100	0.1	0.049	0.018	-0.0016	0.00001	0.00001	0.00001
a_{12}	500	20	3500	3431	3423	3264	3264	3261	3273
v_2 *			0.000299	0.000203	0.000151	0.000312	0.000416	0.000472	0.000429
Constraint Violated				1,5,8	5,8	1,5,11	5	5	5

*v_2 is a function of the above parameters and is computed after the others are estimated.

iterations. In each case, α_1 was adjusted to the limit of its
constraint. Initially, this constraint was 0.5, but this was
changed to 0.43 for purposes of calculating a physically tenable
forward difference for the partial derivative approximation with
a perturbation of 10%. For all iterations, α_5 exceeded α_6 even
after adjustment. After some consideration, it was determined that
this was because the estimation technique was underestimating lock-
up strain but compensating for it by making α_5 greater than α_6. The
next constraint violated is that where $\alpha_7 > \alpha_8$ ($p_1 > p_2$) in the first
two iterations. However, the modified values in iteration 3 and 4
obey the constraint. Also, it is noted that the values obtained for
α_{11} were near zero. This is plausible since the actual yield surface
is a constant and the slope parameter should have no influence upon
it.

In order to calculate the partial derivatives, a forward difference
approximation was employed where the independent variable was per-
turbed by 10 per cent. Since it was believed that such an approxi-
mation may be inaccurate for nonlinear problems, the perturbation
was changed to 1 per cent for the fourth iteration. However, it
was found that the subsequent perturbation was too small for some
of the time responses to show a significant change. Nonetheless,
the resulting state vector of parameters for this iteration indicates
a marked improvement over those obtained in the third iteration.

Figure 7 graphically compares the convergence of the fourth iter-
ation toward the statistical bounds of the actual response. As seen,
the comparison is poorer as one advances toward the negative phase of
the response.

After four iterations AFWL compared the waveforms generated with the
actual curves. The comparison indicated that the process was pro-
ceeding satisfactorily toward convergence and the computational
experiment was deemed to be successfully completed.

While this report was being completed two more iterations were
informally run in conjunction with AFWL. Figure 8 shows a comparison
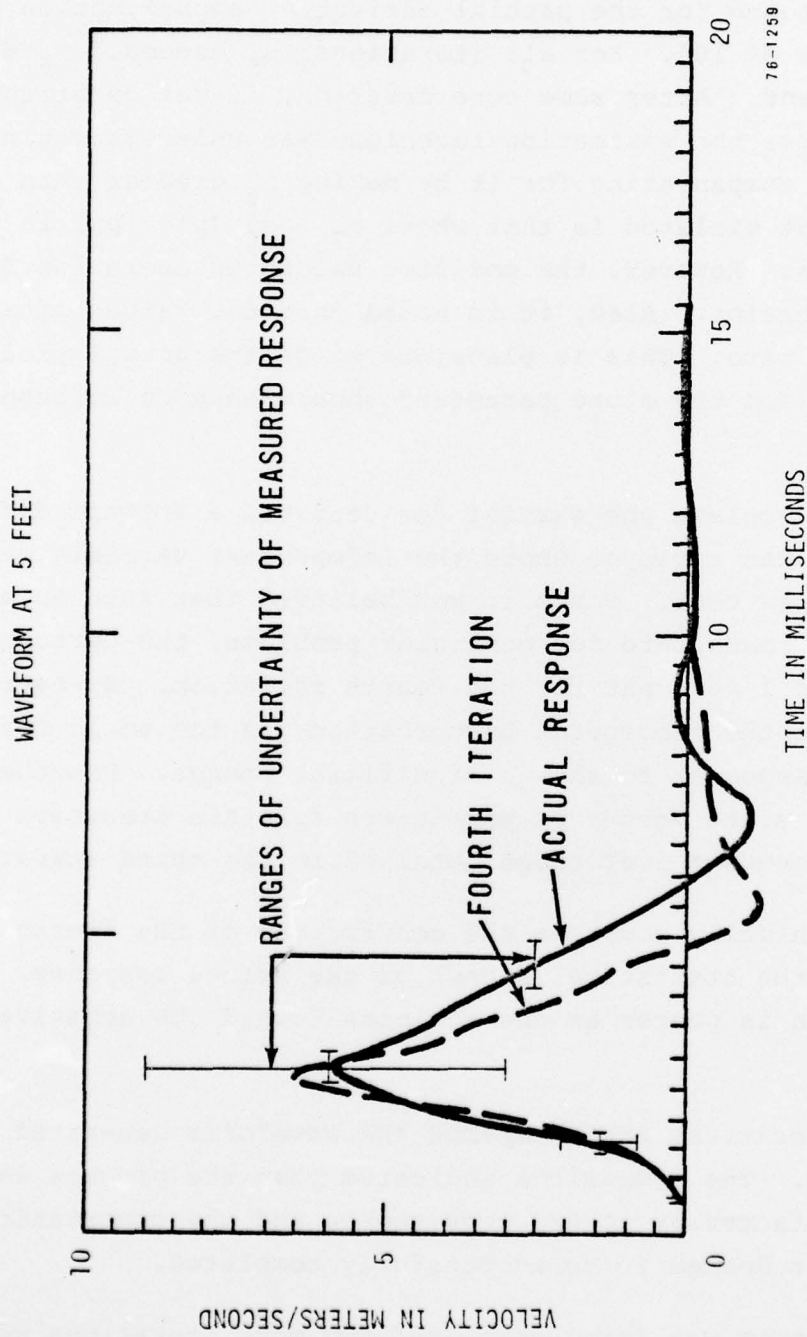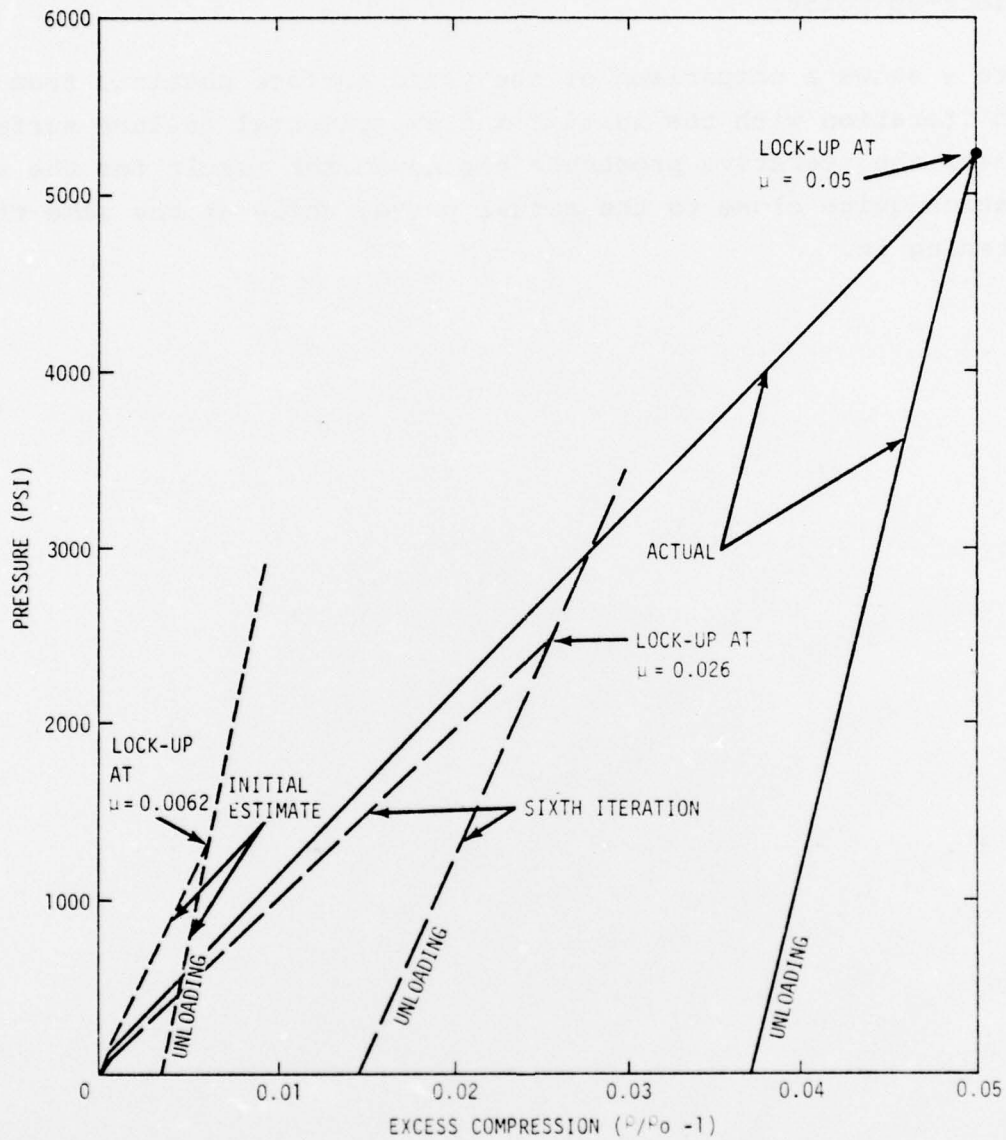of the hydrostat obtained from the sixth iteration with the initial

Figure 7. Convergence of the Iterative Process

Figure 8. Comparison of Theoretical and Experimental Hydrostats

and experimental hydrostats. (The hydrostats for the fourth and
sixth iterations are substantially the same.) As seen, the system
identification process is moving the iterative hydrostat toward the
experiment curve, with good results observed in the loading curve
and lock-up point.

Figure 9 shows a comparison of the yield surface obtained from the
sixth iteration with the initial and experimental failure surfaces.
As seen, the iterative procedure has moved the result for the sixth
iteration quite close to the actual curve, while at the same time
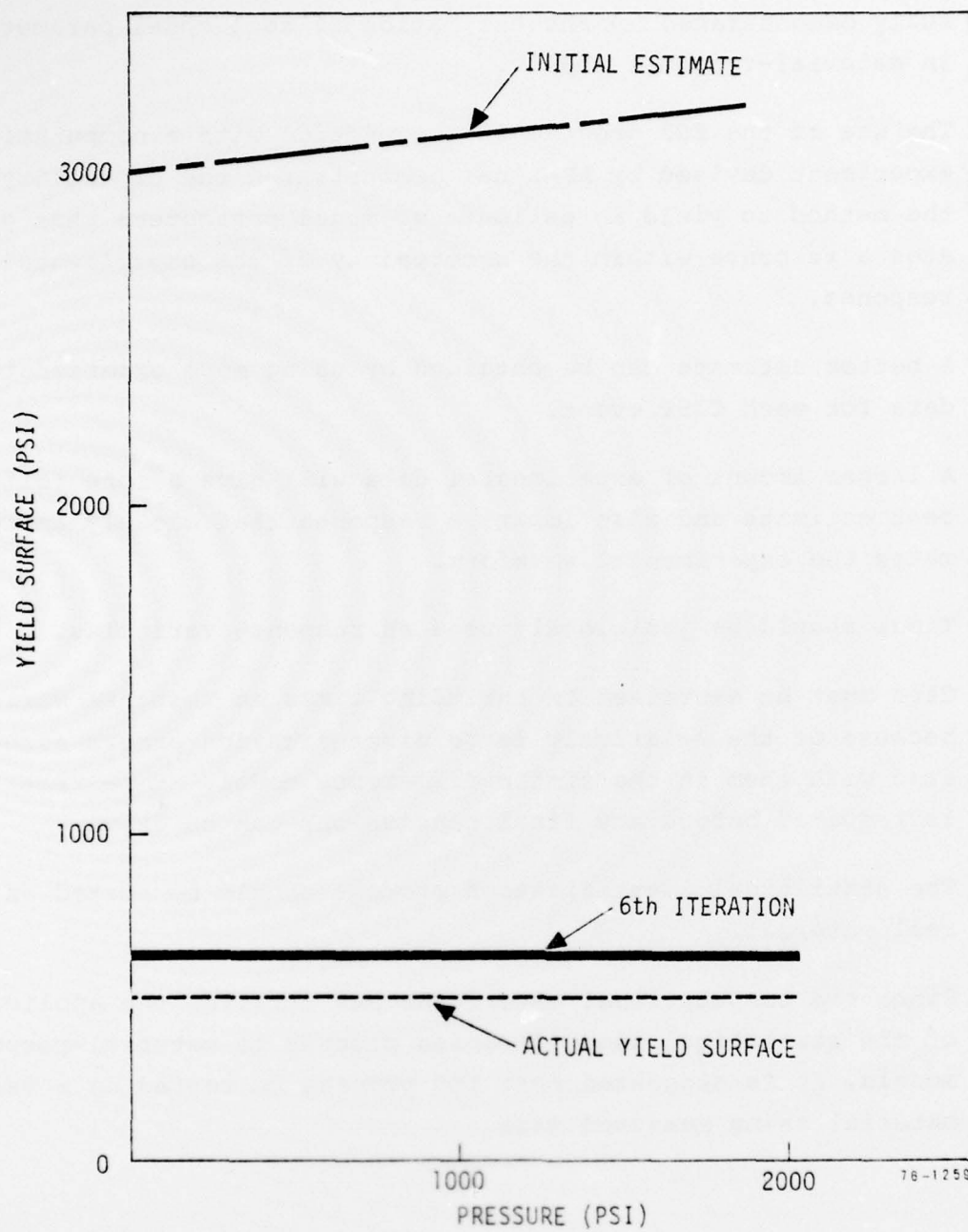flattening it.

Figure 9. Comparison of Yield Surfaces

SECTION IV

CONCLUSIONS AND OBSERVATIONS

(1) The process of statistical identification has been success-
fully demonstrated for the estimation of soil model parameters
in material-response models.

The use of the ESP program in conjunction with a computational
experiment devised by AFWL has demonstrated the capability of
the method to yield an estimate of model parameters that gener-
ates a response within the uncertainty of the experimental
response.

(2) A better estimate can be obtained by using more experimental
data for each CIST event.

A larger amount of experimental data will give a more reliable
best estimate and also impart a response that closely approxi-
mates the experimental waveform.

(3) Times should be judiciously used as response variables.

Care must be exercised in including times as response variables
because of the relatively large discretization errors associ-
ated with them in the finite difference model. More research
is required before any final conclusions can be drawn.

(4) The statistical identification process should be tested on a
real material.

Since the computational experiment has verified the application
of the statistical identification process to material-response
models, it is suggested that the process be tested on a real
material using measured data.